NUMERICAL CALCULATION OF A GENERALIZED THERMAL MODEL OF RADIO-ELECTRONIC APPARATUS

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A method is proposed for numerical calculation of the temperature field of a generalized model of electronic equipment with high component density.

In recent years the literature has presented a lively discussion of methods for developing applied mathematics, and the role of analytical and numerical methods in the realization of mathematical models [1]. In the authors' opinion, the questions thus raised can be answered by a comparative analysis of the possibilities of the various methods employed in the study of complex objects and the processes occurring within such objects. Radio-electronic equipment is an example of such objects, consisting of a system of many bodies with energy sources and drains, the power levels of which change in a complex manner with position and time.

The development of thermal regime studies of electronic equipment has led to creation of generalized thermal and mathematical models [2, 3]. These models consider construction features, distribution of heat producing sources, and the systems used to ensure a normal thermal regime. At the present time computation models have been developed for various special cases of the generalized models, with many of these special cases encompassing a quite wide range of electronic equipment. In the majority of studies the generalized mathematical models have been realized with the aid of approximate analytical methods.

1. Model Description. As was shown in [2, 3], the generalized thermal model of an electronic circuit consists of a heated zone 1 in the form of a rectangular parallelepiped (the circuit functional zone), the chassis 2, in the form of various sets of rectangular plates (each set corresponding to a construction variant of the circuit), with channels 3 in the form of rectangular parallelepipeds, the entry and exit planes of which are located on two opposite faces of the heated zone (Fig. 1). The functional zone is represented as a quasihomogeneous body, the effective thermal conductivity coefficients λ_x of which are defined by the method developed in [2].

We will consider a generalized mathematical model of the electronic circuit. The temperature field of the heated zone is described by a differential equation

$$c_1\rho_1\frac{\partial T_1}{\partial\tau}=\lambda_x\frac{\partial^2T_1}{\partial x^2}+\lambda_y\frac{\partial^2T_1}{\partial y^2}+\lambda_z\frac{\partial^2T_1}{\partial z^2}+q_v(\mathbf{x},\ \tau)-\alpha_v(\mathbf{x})\,[T_1(\mathbf{x},\ \tau)-T_3(\mathbf{x},\ \tau)],\ \mathbf{x}=(x,\ y,\ z) \qquad (1)$$

with boundary condition

$$-\lambda_x\frac{\partial T_1}{\partial n}=\alpha_{12}(\mathbf{x})\,[T_1(\mathbf{x},\ \tau)-T_2(\mathbf{x},\ \tau)]+\alpha_{1c}(\mathbf{x})\,[T_1(\mathbf{x},\ \tau)-T_c]$$
$$+\alpha_{1\text{T}}(\mathbf{x})\,[T_1(\mathbf{x},\ \tau)-T_\text{T}(\mathbf{x})]+q_s(\mathbf{x}). \qquad (2)$$

The form of the equation for the chassis temperature field depends on the chassis form. For definiteness, we will consider a hermetically sealed chassis, using the coordinate system depicted in Fig. 2. In this case the chassis temperature field equation takes on the form

$$c_2\rho_2\frac{\partial T_2}{\partial\tau}=\lambda_2\left(\frac{\partial^2T_2}{\partial x'^2}+\frac{\partial^2T_2}{\partial y'^2}\right)+\beta_{21}(\mathbf{x}')\,[T_2(\mathbf{x}',\ \tau)-T_1(\mathbf{x}',\ \tau)]-$$
$$-\beta_{2c}(\mathbf{x}')\,[T_2(\mathbf{x}',\ \tau)-T_c]-\beta_{2\text{T}}(\mathbf{x}')\,[T_2(\mathbf{x}',\ \tau)-T_\text{T}(\mathbf{x}')]+q(\mathbf{x}'), \qquad (3)$$
$$\mathbf{x}'=(x',\ y').$$

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The boundary conditions for Eq. (3) in the region shown in Fig. 2 consist of conditions of the fourth type for points which coincide with each other when the region is "folded" into a rectangular parallelepiped.

The equation for the cooling agent flowing in the i-th channel in the direction of the axis Oy (Fig. 1) has the form

$$c_3\rho_3 \frac{\partial T_3}{\partial \tau} + c_3\rho_3 U^i \frac{\partial T_3}{\partial y} = \alpha_v(\mathbf{x})\,[T_1(\mathbf{x},\ \tau) - T_3(\mathbf{x},\ \tau)],\ i = 1,\ 2,\ \ldots,\ p. \tag{4}$$

For Eq. (4) the cooling agent temperature is usually specified at the channel entrance

$$T_3(x,\ 0,\ z,\ \tau) = T_3^i\ \text{in}. \tag{5}$$

Initial conditions for Eqs. (1), (3), (4) are

$$T_1(\mathbf{x},\ 0) = T_{10}(\mathbf{x}),\ T_2(\mathbf{x},\ 0) = T_{20}(\mathbf{x}),\ T_3(\mathbf{x},\ 0) = T_{30}^i. \tag{6}$$

We will note the features of the model of Eqs. (1)-(6) which distinguish it from the general model of [3]: a) the linear variant of the problem is considered; b) a surface heat drain model with effective heat-exchange coefficient and temperature is used to describe the action of the cooling elements; c) the effective cooling element parameters are considered constant in time.

2. Numerical Calculation of Model. The literature on numerical methods has presented a thorough study of one parabolic-type equation for a region of complex form [4]. In the present case one unknown function appears in the boundary conditions for the other function. Insufficient attention has been given to such systems of equations, and there are few recommendations for constructions of difference schemes for their solution. Therefore, in constructing an approach to solution of system (1)-(6) it is desirable to turn to results of numerical experiments and consider difficulties in program realization.

After analysis, the following difference pattern was chosen. We assume that at the j-th step in time the grid functions corresponding to T_1, T_2, T_3 are found (at j = 0 they are calculated with T_{10}, T_{20}, T_{30}). Their calculation in the (j + 1)-th step begins with a calculation of the heated zone temperature field. In its determination, the temperature fields for the chassis and cooling agent are taken from the preceding j-th step. Then Eq. (1) can be considered as an equation for a solitary body, and the corresponding techniques applied. For the heated zone the local-one-dimensional technique of [4] was used. Thereupon, the grid function determined for the heated zone temperature field is used to calculate the temperature fields of the chassis and cooling agent. The local-one-dimensional technique was used for the body, while for the cooling agent an implicit technique with second-order approximation in the spatial variable was employed.

The numerical method described above was realized as a FORTRAN IV program for the ES-1022 computer, the program being designed for solution of the general model of Eqs. (1)-(6). However, provision was made for examination of a number of special cases of practical importance.

In the study of the numerical method described it is important to determine the machine time expenditures required to obtain results with required accuracy. To do this, calculations of several typical models of electronic circuits were performed.

Depending on the method used to determine error in the numerical calculation, three groups of models are desirable: those admitting an exact analytical solution; those for which experimental data are available; and those for which there exist neither an exact analytic solution nor experimental data.

In the first case, the accuracy was evaluated by comparison with the exact analytical solution; in the second, by comparison with the experimental data; and in the third, by Runge's rule. It should be noted that in the second case the difference between calculation and experiment is also produced by inaccuracy of the model used and the original information.

All three groups were studied.

Model Permitting Exact Analytical Solution. For this study a special case of Eqs. (1)-(6) was taken, in which the thermal model consists of a single heated zone with no heat drains, the equation for which appears in dimensionless form as:

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = k_{\bar{x}}^2\varepsilon_{\bar{x}}\frac{\partial^2\theta}{\partial \bar{x}^2} + k_{\bar{y}}^2\varepsilon_{\bar{y}}\frac{\partial^2\theta}{\partial \bar{y}^2} + \frac{\partial^2\theta}{\partial \bar{z}^2} + 1, \tag{7}$$

$$\left(-\frac{\partial\theta}{\partial n} + \mathrm{Bi}_m\theta\right)_{m=\bar{x},\bar{y},\bar{z}} = 0, \tag{8}$$

$$\theta(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})|_{\mathrm{Fo}=0} = 0, \tag{9}$$

where $\bar{x} = x/L_x$; $\bar{y} = y/L_y$; $\bar{z} = z/L_z$; $\mathrm{Fo} = a_z\tau/L_z^2$; $\varepsilon_m = \lambda_m/\lambda_z$; $k_m = L_z/L_m$; $\mathrm{Bi}_m = \alpha_{1c}L_m/\lambda_m$; $\theta = (T_1 - T_c)\lambda_z/q_v L_z^2$; $m = x, y, z$.

The accuracy of the numerical technique was analyzed over ranges of the parameters k_m, ε_m, Bi_m, characteristic of real electronic equipment, and for Fo values ranging from zero to values corresponding to a stationary regime. Parameter values were taken within the following intervals: $k_{\bar{x}} = k_{\bar{y}} = k \in [1;\ 2]$, $\varepsilon_{\bar{x}} = \varepsilon_{\bar{y}} = \varepsilon \in [0.5;\ 50]$, $\mathrm{Bi}_{\bar{x}} = \mathrm{Bi}_{\bar{y}} = \mathrm{Bi}_{\bar{z}} = \mathrm{Bi} \in [2.5;\ 50]$.

The error of the numerical method was determined by comparison with the exact analytical solution of Eqs. (7)-(9) $\theta_t(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})$, obtained by the method of finite-integral transforms [5].

The error was characterized by two quantities:

$$\delta_1 = \max_{\bar{x},\bar{y},\bar{z},\mathrm{Fo}} |\delta_1(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})| = \max_{\bar{x},\bar{y},\bar{z},\mathrm{Fo}} \left| \frac{\theta(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo}) - \theta_T(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})}{\theta_T(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})} \right|, \tag{10}$$

$$\delta_2 = \max_{\bar{x},\bar{y},\bar{z},\mathrm{Fo}} |\delta_2(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})| = \max_{\bar{x},\bar{y},\bar{z},\mathrm{Fo}} \left| \frac{\theta(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo}) - \theta_T(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})}{\theta_{T\max}(\mathrm{Fo})} \right|,$$

where

$$\theta_{T\max}(\mathrm{Fo}) = \max_{\bar{x},\bar{y},\bar{z}} |\theta_T(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})|. \tag{11}$$

The reduced error δ_2 is used because at high Bi the levels of the temperatures θ_t and θ near the boundaries of the region are low, and the small difference between them may prove comparable to θ_t. Therefore, the relative error $\delta_1(\bar{x}, \bar{y}, \bar{z}, \mathrm{Fo})$ obtained in this case is significant. Thus, one could conclude that the numerical method is of low quality, although in practice at low temperature levels absolute errors comparable in magnitude to those temperatures play no significant role.

Analysis revealed that the relative error did not exceed 5% over the entire range of k, ε, Bi studied. With increase in Bi δ_1 increased, reaching 30% at Bi = 50. The computation time on the ES-1022 computer was no longer than 6 min.

Circuit Models for Which Experimental Data Are Available. A circuit constructed in motherboard form was used for the study [6]. The device consisted of a motherboard base with five circuitboards all of the same size installed therein. The five circuits had differing internal heat-source distributions. The outer surface of the device dissipates heat by radiation and convection into the surrounding medium.

The thermal model of the device described (Fig. 3) is a homogeneous anisotropic parallelepiped. Heat exchange with the external medium, the temperature of which is assumed constant in time, is described by type third boundary conditions with heat-exchange coefficients identical for all boundaries. The volume heat-source distribution is nonuniform. The differential equation for the model temperature field has the form

$$c_1\rho_1\frac{\partial T_1}{\partial\tau} = \lambda_x\frac{\partial^2 T_1}{\partial x^2} + \lambda_y\frac{\partial^2 T_1}{\partial y^2} + \lambda_z\frac{\partial^2 T_1}{\partial z^2} + q_v(\mathbf{x}) \tag{12}$$

with boundary conditions

$$-\lambda(\mathbf{x})\frac{\partial T_1}{\partial n} = \alpha_{1c}(T_1 - T_c),\ T_1(\mathbf{x}, 0) = T_c,\ \mathbf{x} = (x, y, z). \tag{13}$$

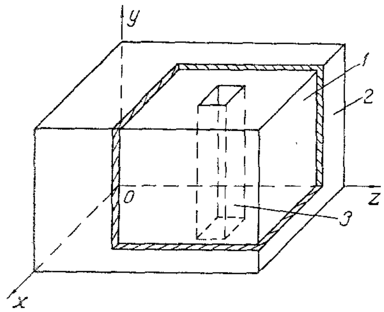

Fig. 1

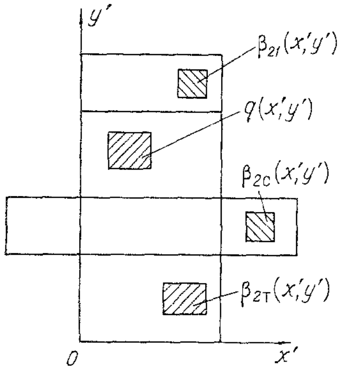

Fig. 2

Fig. 1. Generalized thermal model of electronic circuit: 1) heated zone; 2) chassis; 3) cooling agent channel.

Fig. 2. Chassis thermal model: $q(x', y')$, heat-source distribution, W/m^3; $\beta_{21}(x', y')$, $\beta_{2T}(x', y')$, $\beta_{2C}(x', y')$, distribution of chassis volume heat-exchange coefficients with heated zone, heat-exhaust elements, and medium, W/m^3•deg.

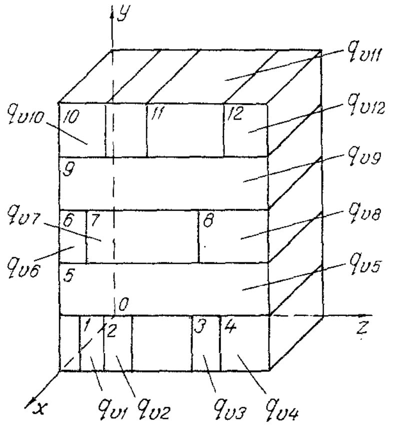

Fig. 3

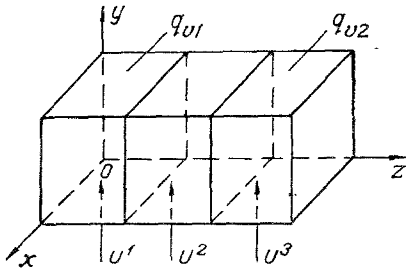

Fig. 4

Fig. 3. Thermal model of electronic device with motherboard construction: q_{vi}, volume heat source distribution, W/m^3, i = 1,...,12.

Fig. 4. Thermal model of electronic device with forced cooling: U^i, velocity of cooling agent in i-th channel, m/sec; i = 1, 2, 3.

It is evident that the problem of Eqs. (12), (13) is a special case of the generalized mathematical model of Eqs. (1)-(6).

The accuracy of the numerical method was analyzed by comparing the computation results with results of the experimental study [6] by determining the values of δ_1 and δ_2 with Eqs. (10), (11). Analysis revealed that the numerical method permitted calculation of the temperature field of the device described with an accuracy suitable for engineering calculations. The values of δ_1 and δ_2 did not exceed 15%. Calculation time on the ES-1022 was less than 8 min.

Models with Exact Analytic Solution or Experimental Data. A special case of Eqs. (1)-(6) was chosen, in which the thermal model of the electronic device (Fig. 4) was a system of two bodies, a heated zone with cooling agent channels passing through it. The heated zone is in the form of a homogeneous anisotropic parallelepipid containing three regions with heat sources of differing specific power level. Convective motion of cooling air flows occurs through channels in the direction of the axis Oy within the heated zone. The boundaries of the three perforation regions coincide geometrically with the boundaries of the three heat-producing regions.

The temperature field of the device is described by the following system of equations:

$$c_1\rho_1 \frac{\partial T_1}{\partial \tau} = \lambda_x \frac{\partial^2 T_1}{\partial x^2} + \lambda_y \frac{\partial^2 T_1}{\partial y^2} + \lambda_z \frac{\partial^2 T_1}{\partial z^2} + q_v(z) - \alpha_v(\mathbf{x})[T_1(\mathbf{x}, \tau) - T_3(\mathbf{x}, \tau)], \tag{14}$$

$$-\lambda(\mathbf{x}) \frac{\partial T_1}{\partial n} = \alpha_{1c}(\mathbf{x})[T_1(\mathbf{x}, \tau) - T_c], \tag{15}$$

$$T_1(\mathbf{x}, 0) = T_c, \tag{16}$$

$$c_3\rho_3 \frac{\partial T_3}{\partial \tau} + c_3\rho_3 U^i \frac{\partial T_3}{\partial y} = \alpha_v^i [T_1(\mathbf{x}, \tau) - T_3(\mathbf{x}, \tau)], \tag{17}$$

$$T_3(x, 0, z, \tau) = T^i_{3\mathrm{in}},\ i = 1, 2, 3, \tag{18}$$

$$T_3(\mathbf{x}, 0) = T^i_{30}. \tag{19}$$

The computer program was used to calculate the nonstationary temperature field of the model of Eqs. (14)-(19).

Due to the absence of an exact solution of Eqs. (14)-(19) and the lack of experimental data, the accuracy of the numerical method was determined by Runge's rule. Analysis of the calculation results showed that the relative error calculated by Eq. (10) does not exceed 3% (3% for the heated zone, 2% for the cooling agent). The time required by the ES-1022 until the system reached a practically stationary thermal regime was on the order of 5 min.

3. Evaluation of the Results. The results presented as to accuracy of the proposed method permit the preliminary conclusion that it is effective. In all problems considered, it was possible to achieve an accuracy satisfactory for engineering calculations. For 300 computation points the error lay in the range 2-10% for a computation time of less than 10 min.

In performing the calculations, the nonstationary temperature field of the entire device was obtained. Therefore, further study of more complex generalized models is desirable, considering the nonlinearity of real problems, a larger number of heated zones, etc., as well as generalized models of other classes of electronic equipment.

Moreover, it would be desirable to study the possibilities of the numerical method in performing mathematical experiments, the goal of which would be to determine the effect of various parameters on temperature of the most critical regions of the electronic device, upon which the results should be generalized and placed in convenient form.

NOTATION

x, y, z, x', y', spatial coordinates, m; τ, time, sec; L_x, L_y, L_z, dimensions of heated zone, m; λ_x, λ_y, λ_z, effective thermal-conductivity coefficients of heated zone, W/m•deg; λ_2, thermal conductivity of chassis, W/m•deg; a_z, thermal diffusivity of heated zone along z axis, m^2/sec; c_1, effective specific heat of heated zone, J/kg•deg; ρ_1, effective density of heated zone, kg/m^3; c_3, ρ_3, c_2, ρ_2, thermophysical characteristics of cooling agent and chassis, J/kg•deg•kg/m^3; $q_v(x, \tau)$, $q(x', y')$, volume heat-source distribution, W/m^3; $q_s(x)$, surface heat-source distribution, W/m^2; p, number of cooling agent channels; Fo, Fourier number; Bi, Biot number; U^i, coolant velocity in i-th channel, m/sec; $T_1(x, \tau)$, $T_2(x, \tau)$, $T_3(x, \tau)$, temperature distribution of heated zone, chassis, and coolant, °K; T_{30}, $T_{10}(x)$, $T_{20}(x)$, initial temperatures, °K; T_{3in}, coolant temperature at input to channel, °K; $T_T(x)$, effective temperature distribution of heat loss elements, °K; T_C, temperature of external medium; °K; θ, dimensionless heated zone temperature; $\alpha_v(x)$, local volume heat exchange coefficient, W/m^3•deg; $\alpha_{12}(x)$, $\alpha_{1C}(x)$, $\alpha_{1T}(x)$, heat liberation coefficients; W/m^2•sec; $\beta_{21}(x', y')$, $\beta_{2C}(x', y')$, $\beta_{2T}(x', y')$, volume heat-exchange coefficients of chassis with heated zone, medium, and cooling elements, W/m^3•deg.

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APPROXIMATING THE EFFECTS OF THERMAL PULSE ACTION ON A METAL BY GENERALIZING THE DIAGRAM OF PHASE-BOUNDARIES DISPLACEMENT

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A surface thermal pulse excites in a metal a wave of phase transformations whose mathematical description requires a formulation of strongly nonlinear thermophysical problems [1]. A machine solution of such problems is difficult because of the large volume of calculations, even for individual pulse modes, but the task becomes still more unwieldy when numerical data covering many metals over a wide range of pulse modes are needed.

In this respect it is important that the results of computer-aided numerical solution of those thermophysical problems can be analytically generalized and on this basis, as has been shown in an earlier study [2], a system of equations can be proposed which will approximate, within an acceptable degree of accuracy, the final one-dimensional displacement of phase boundaries in a metal due to action of a thermal pulse. This makes it possible to use those equations for obtaining extensive information about many metals and pulse modes without going through a numerical solution of the actual thermophysical problems, which naturally deserves to be carefully considered.

In the earlier study [2] there was proposed a method of using those equations for constructing the diagram of the final displacement of phase boundaries in a metal due to action of a surface thermal Π-pulse with a given surface energy density W and a variable action time t. In this study the problem will be considered in broader terms, viz. constructing a generalized diagram of the final displacement of phase boundaries in a metal with both parameters W and t of a thermal pulse varied.

The generalized diagram will be calculated and constructed so that it will describe, with sufficient accuracy, the quantitative relation between the main parameters of a thermal pulse W, t and the main results of its action on a metal. The effect of a thermal pulse can, moreover, be characterized by displacement of the melting front $y_m = y_m(W, t)$ or by displacement of the evaporation front $y_e = y_e(W, t)$, or by the relative displacement of the evaporation front α

$$\alpha = \frac{y_e}{y_m};\ \alpha = \alpha(W,\ t);\ 0 \leqslant \alpha \leqslant 1. \tag{1}$$

A search for the optimum variant of this generalized diagram has revealed that it is most expediently drawn in the form of the relation between four quantities

$$\Phi = \Phi(W,\ t,\ y_m,\ \alpha). \tag{2}$$

The main difficulty in calculations for the generalized diagram is related to the need to find the roots of transcendental equations, which requires appropriate numerical methods of solution. These authors have developed an algorithm of calculations for the complete generalized diagram realizable on a small computer. It is based on the diagram representing the re-

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